A SAT-Based Approach to Cooperative Path-Finding Using All-Different Constraints

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Abstract

The approach to solving cooperative-path finding (CPF) as satisfiability (SAT) is revisited. An alternative encoding that exploits multi-valued state variables representing locations where a given agent resides is suggested. This encoding employs the ALL-DIFFERENT constraint to model the requirement that agents must not collide with each other. We show that our new domain-dependent encoding enables finding of optimal or near optimal solutions to CPFs in certain hard setups where A*-based techniques such as WHCA* fail to do so. Our finding is also that the ALL-DIFFERENT encoding can be solved faster than the existent encoding.

Introduction and Context

The problem of *cooperative path-finding* (CPF) (Silver, 2005; Ryan, 2008; Surynek, 2009) consists in finding non-colliding spatial-temporal paths for agents that need to relocate themselves from given *initial* locations to given *goal* locations. A generally adopted abstraction is that the environment is modeled as an undirected graph with agents placed in its vertices. At most one agent is placed in a vertex and at least one vertex remains unoccupied. The move is possible along an edge into a currently unoccupied vertex (an example instance of CPF on a 4-connected grid is shown in Figure 1).

In our work we addressed the case of **optimal** or **near optimal makespan** and **densely populated** environments. We employ the SAT solving technology to optimize the makespan of solutions generated by existent fast sub-optimal techniques such as BIBOX (Surynek, 2009) or PUSH-SWAP (Luna & Berkis, 2011). In contrast to the approach adopted in *domain independent* SAT-based planning (Kautz and Selman, 1999; Huang et al., 2010) we do not encode the whole problem as a SAT instance but only sub-problems represented by subsequences of the original solution are encoded. These (sub-optimal) sub-solutions are subsequently replaced by optimal ones found by the SAT solver. The similar approach has been recently applied in domain-independent planning by Barták, Balyo, and Surynek (2012). We also propose a new compact domain dependent encodings for CPFs – called ALL-DIFFERENT encoding – as an alternative to domain independent encodings used in SAT-based planning and to the encoding proposed in (Surynek, 2012).

Cooperative Path-Finding (CPF) Formally

An arbitrary **undirected graph** can be used to model the environment where agents are moving. Let $G = (V, E)$ be such a graph where $V = \{v_1, v_2, \dots, v_n\}$ and $E \subseteq \binom{V}{2}$. The placement of agents in the environment is modeled by assigning them vertices of the graph. Let $A = \{a_1, a_2, \dots, a_\mu\}$ be a finite set of *agents*. Then, an arrangement of agents in vertices of graph G will be fully described by a *location* function $\alpha: A \longrightarrow V$; the interpretation is that an agent $a \in A$ is located in a vertex $\alpha(a)$. At most **one agent** can be located in each vertex; that is α is uniquely invertible.

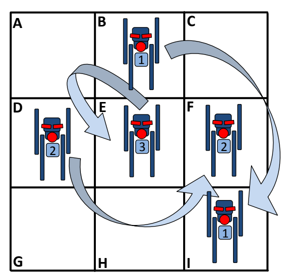

Figure 1. *An instance of CPF.* Three agents need to relocate themselves in the 4-connected grid 3×3.

Definition 1 (COOPERATIVE PATH FINDING). An instance of *cooperative path-finding* problem is a quadruple $\Sigma = [G = (V, E), A, \alpha_0, \alpha_+]$ where location functions α_0 and α_+ define the initial and the goal arrangement of a set of agents A in G respectively. □

An arrangement α_i at the i-th time step can be transformed by a transition action which instantaneously moves agents in the non-colliding way to form a new arrangement α_{i+1}. The resulting arrangement α_{i+1} must satisfy the following *validity conditions*:

(i) $\forall a \in A$ either $\alpha_i(a) = \alpha_{i+1}(a)$ or $\{\alpha_i(a), \alpha_{i+1}(a)\} \in E$ holds,

(ii) $\forall a \in A \;\; \alpha_i(a) \neq \alpha_{i+1}(a) \Rightarrow \alpha_i^{-1}(a) = \perp$, and

(iii) $\forall a, b \in A \;\; a \neq b \Rightarrow \alpha_{i+1}(a) \neq \alpha_{i+1}(b)$.

The task in cooperative path finding is to transform α_0 using above valid transitions to α_+.

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CPF as Propositional Satisfiability

To enable solving of CPF as satisfiability we needed to develop compact SAT encodings. We followed the classical *Graphplan* style. We choose the location function to represent state variables. Hence we need to take care of ensuring validity conditions (ii) and (iii) explicitly. An agent must move into unoccupied vertex which means it should avoid all the vertices occupied by other agents. This condition is modeled by pair-wise differences between involved location state variables. At the same time, it is necessary that no two agents occupy the same vertex (location). This requirement can be expressed through the ALL-DIFFERENT constraint involving all the location state variables at the given time step. The advantage of using domain-dependent encoding is illustrated in Table 1.

Definition 2 (REGULAR LAYER – ALL-DIFFERENT). The i-th layer of the *ALL-DIFFERENT encoding* consists of the following finite domain integer **state variables**:

- $\mathcal{L}_i^a \in \{0,1,2,\dots,n\}$ for all $a \in A$
  such that $\mathcal{L}_i^a = l$ iff $\alpha_i(a) = v_l$

and the **constraints** are as follows:

- for all $a \in A$ and $l \in \{1,2,\dots,n\}$
  $$\mathcal{L}_i^a = l \Rightarrow \mathcal{L}_{i+1}^a = \ell \vee \bigvee_{\ell \in \{1,\dots,n\} \mid \{v_l, v_\ell\} \in E} \mathcal{L}_{i+1}^a = \ell$$
  (agents can move only **along edges** of G),
- for all $a \in A$
  $$\bigwedge_{b \in A \mid b \neq a} \mathcal{L}_{i+1}^a \neq \mathcal{L}_i^b$$
  (the **target vertex** of agent's move must be **empty**),
- and **at most one** agent resides in each vertex:
  $$\mathrm{AllDifferent}(\mathcal{L}_i^{a_1}, \mathcal{L}_i^{a_2}, \dots, \mathcal{L}_i^{a_\mu})$$

which altogether directly encodes validity conditions (i), (ii), and (iii). □

Table 1. *Encoding sizes comparison on the grid* ***8×8***. The number of layers of encodings was determined as the goal level provided by SATPLAN (a step where the goal may be reachable).

$\|A\|$ in the 4-connected grid **8×8**	Number of layers	SATPLAN encoding		SASE encoding		ALL-DIFFERENT encoding	
		|Variables|	|Clauses|	|Variables|	|Clauses|	|Variables|	|Clauses|
8	8	10022	165660	19097	105724	25136	114952
16	10	30157	1169198	51662	372140	79008	326736
32	14	99398	8530312	157083	1385010	309824	1120672

SAT-Based Optimization of Solutions to CPFs

The approach of our choice is to partition a given suboptimal solution to CPF called a *base solution* into relatively small pieces. Each of these pieces is then replaced by the optimal solution found by the SAT solver. The process is iterated with the newly obtained solution as the base until no improvement can be made. The process is shown in Figure 2. The original base solution is generated by the BIBOX algorithm (Surynek, 2009).

To evaluate the benefit of the proposed approach we made a brief comparison against WHCA* (Silver, 2005) on a 4-connected 8×8 grid with random initial and goal arrangements of agents – see Figure 2.

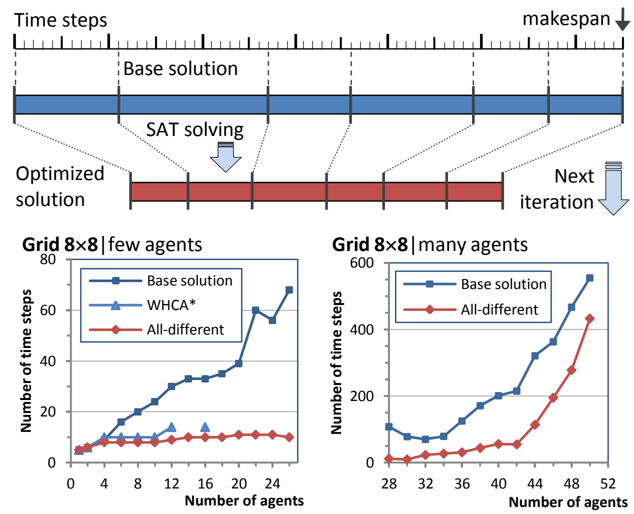

Figure 2. *Illustration of the* ***optimization process*** and ***makespan*** *comparison on the 8×8 grid.* Optimal solutions for up to 22 can be found. Only up to 16 agents can be solved by WHCA*.

We demonstrate our approach to be able to solve more instances and generate shorter solutions than WHCA*. Another not presented experiment indicates that the ALL-DIFFERENT dominates over the encoding from (Surynek, 2012) in the case of sparsely populated environments.

For future work we plan to enhance the ALL-DIFFERENT encoding with filtering of unreachable locations.

References

Barták, R., Balyo, T., Surynek, P., 2012. *On Improving Plan Quality via Local Enhancements*. Proceedings of SOCS 2012, to appear.

Huang, R., Chen, Y., Zhang, W., 2010. *A Novel Transition Based Encoding Scheme for Planning as Satisfiability.* Proceedings AAAI 2010, AAAI Press.

Kautz, H., Selman, B., 1999. *Unifying SAT-based and Graph-based Planning.* Proceedings of IJCAI 1999, pp. 318-325, Morgan Kaufmann.

Luna, R., Berkis, K., E., 2011. *Push-and-Swap: Fast Cooperative Path-Finding with Completeness Guarantees.* Proceedings of IJCAI 2011, pp. 294-300, IJCAI/AAAI Press.

Ryan, M. R. K., 2008. *Exploiting Subgraph Structure in Multi-Robot Path Planning.* JAIR, Volume 31, pp. 497-542, AAA Press.

Silver, D., 2005. *Cooperative Pathfinding.* Proceedings of AIIDE 2005, pp. 117-122, AAAI Press.

Surynek, P., 2009. *A Novel Approach to Path Planning for Multiple Robots in Bi-connected Graphs.* Proceedings of ICRA 2009, pp. 3613-3619, IEEE Press.

Surynek, P., 2012. *Towards Optimal Cooperative Path Planning in Hard Setups through Satisfiability Solving.* Proceedings of PRICAI 2012, to appear.