A SAT-Based Approach to Cooperative Path-Finding Using All-Different Constraints

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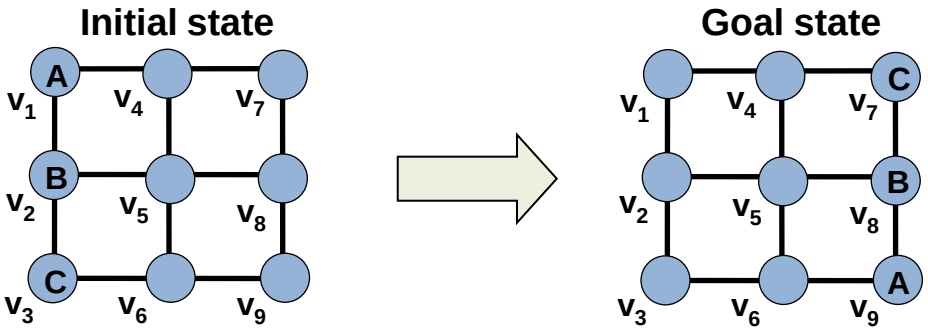
Cooperative Path-finding (CPF)

- plan movements of agents in **space** and **time**
  - **time** – discrete ⇒ time steps
  - **space** – abstract ⇒ graph G=(V,E)
- **requirements**
  - all agents reach a given **goal vertex**
  - agents do **not collide** with each other

    (move only to vacant vertices)



Set of **agents** = {1,2,3}

plan for **agent A** = [$\mathbf{v_1}$, v_4, v_7, v_8, v_9, v_9, $\mathbf{v_9}$]
plan for **agent B** = [$\mathbf{v_2}$, v_2, v_1, v_4, v_7, v_8, $\mathbf{v_8}$]
plan for **agent C** = [$\mathbf{v_3}$, v_3, v_3, v_2, v_1, v_4, $\mathbf{v_7}$]

Time step: **1 2 3 4 5 6 7**

makespan = 7

Current Techniques / Our Approach

fast, complete	relatively fast	optimal makespan
✕ long makespan	✕ incomplete	✕ slow
polynomial time sub-optimal	search based sub-optimal	search based optimal

\+ **SAT** Solver

\+ **encoding** of CPF

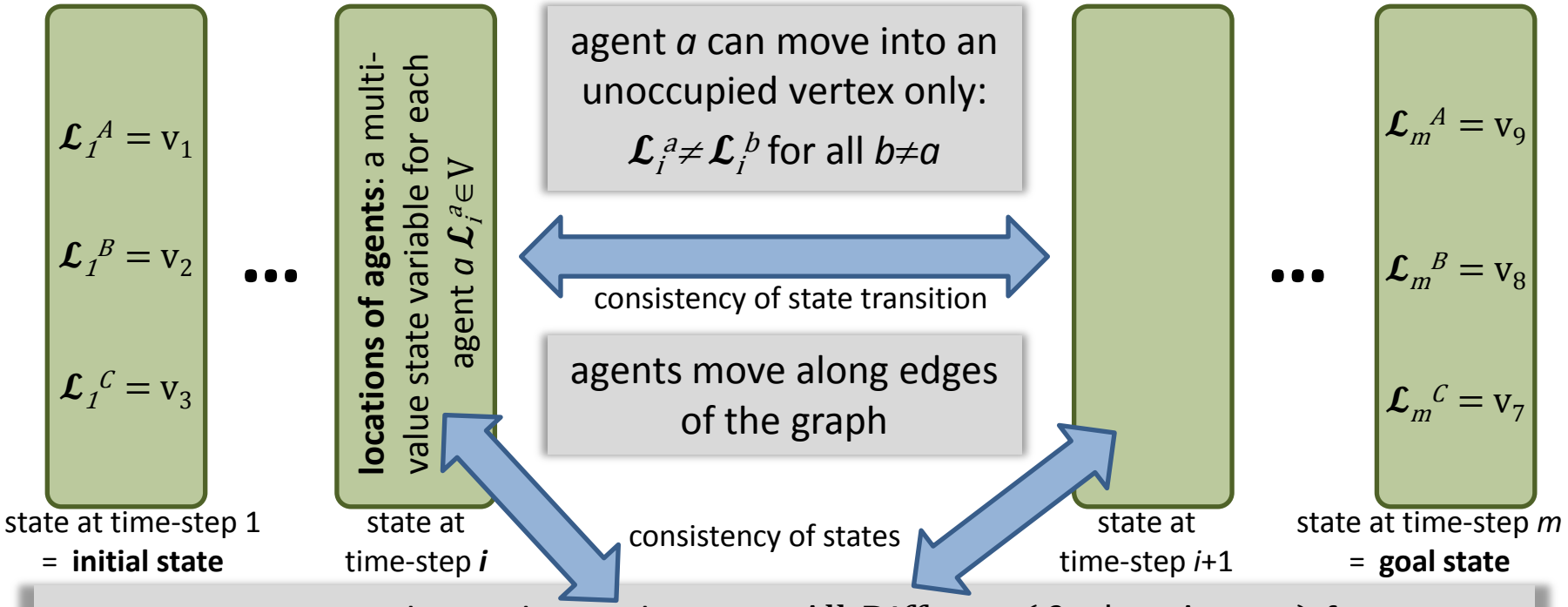
\+ optimization **strategy**

= **Our new approach – iCOBOPT**

- (quickly) find sub-optimal solution
- replace sub-sequences with makespan-optimal sub-solutions
- repeat the process

SAT Encoding of CPF

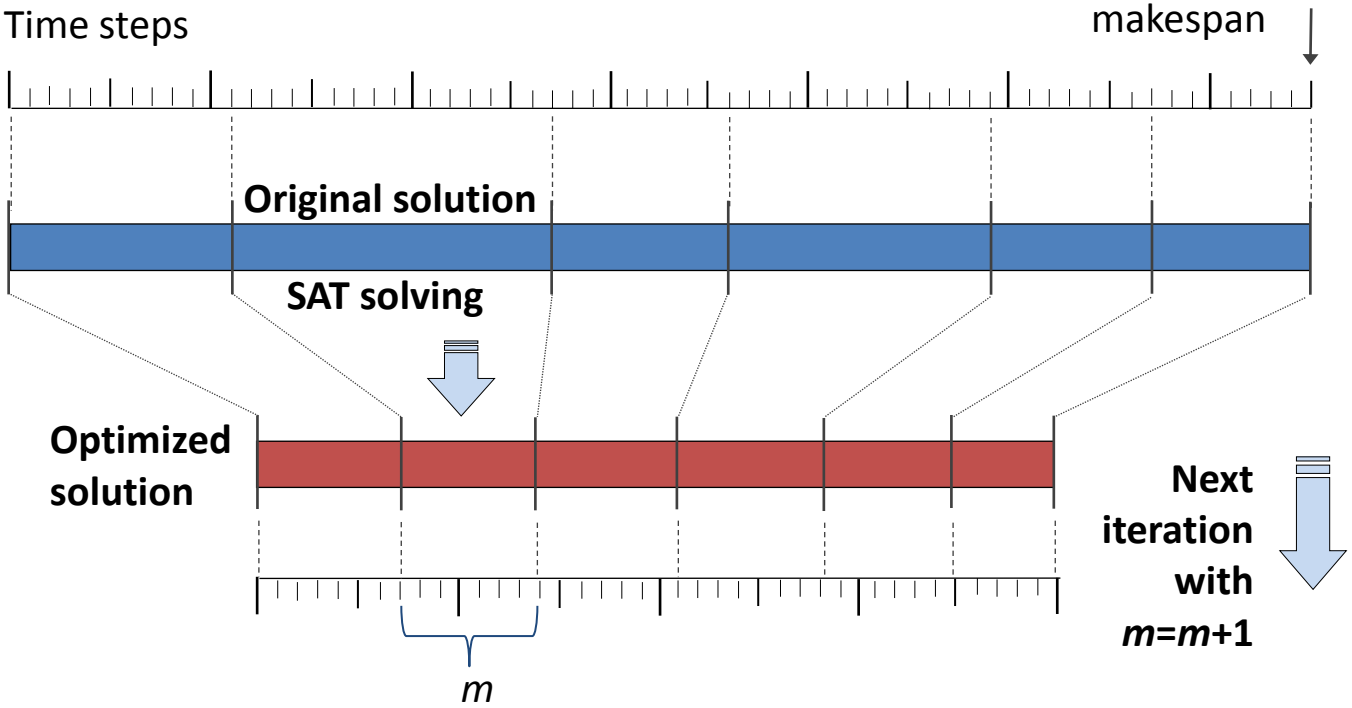
- encoding for the **fixed makespan *m***
- encode state **at each time-step**
  - multi-value state variables $\Rightarrow$ bit-vectors



at most one agent is located in each vertex: All-Different($\mathcal{L}_i^a$ | $a \in$ Agents) for i=1,…,m

Optimization Strategy - iCOBOPT

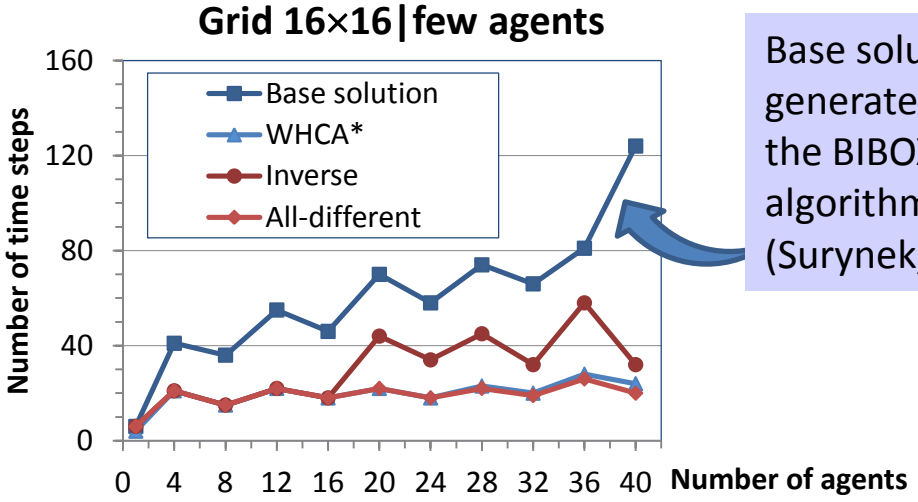
- for a fixed makespan *m* find the longest sub-sequence of the original solution that can be replaced with corresponding optimal sub-solution of makespan *m*



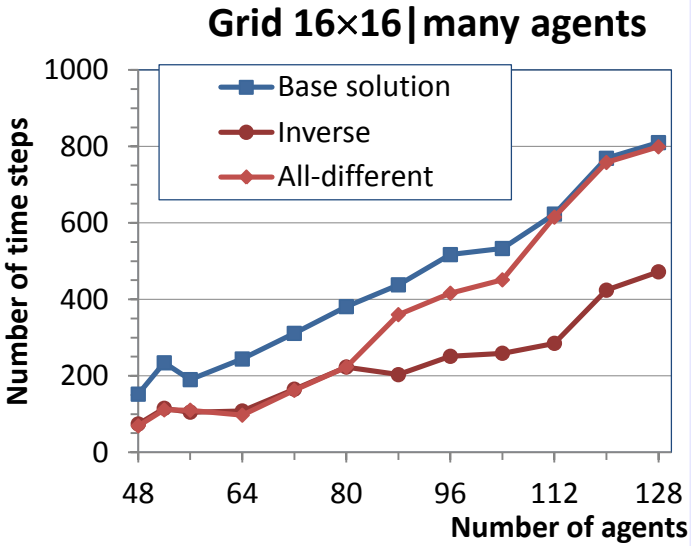
- iCOBOPT is similar to turbo-fullstep

Experimental Results and Comparison

- setup: G=(V,E) = **4-connected grid**
  random initial and goal arrangement of agents



Base solutions generated by the BIBOX algorithms (Surynek, 2009)



Number of agents	4-connected grid 16x16		
	Optimal makespan	SATPLAN Runtime (s)	SASE Runtime (s)
1	4	0.68	1.66
4	21	195.5	17.98
8	15	1396.07	128.87
16	N/A	Out of memory	Timeout

Number of agents	4-connected grid 16x16		
	Computed makespan	INVERSE Runtime (s)	ALL-DIFF Runtime (s)
1	6/6	0.074	0.070
4	21/21	319.785	45.367
8	15/15	152.625	62.955
16	18/18	1833.080	910.391

Conclusions and Related Works

- Good performance on graphs with dense population agents
- Sometimes optimal solution can be found
- Encoding, sub-optimal algorithm, and optimization strategy can be improved independently

References

Balyo, B., Barták, R., Surynek, P. 2012. *On Improving Plan Quality via Local Enhancements*, SoCS 2012, AAAI Press.

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