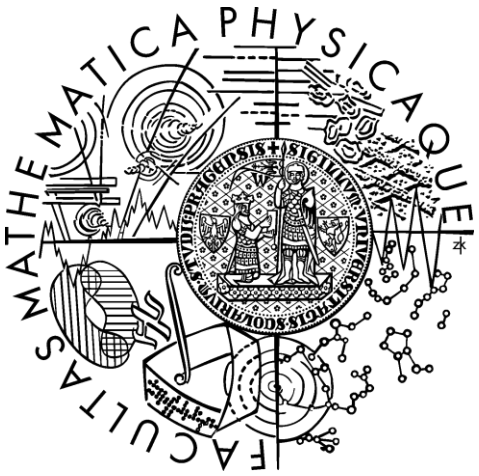

Near Optimal Cooperative Path Planning in Hard Setups through Satisfiability Solving

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Problem of Cooperative Path-planning (CPP)

- **Abstraction** for tasks of motion of multiple (autonomous or passive) entities in a certain environment (real or virtual).
  - Entities are given an **initial** and a **goal** arrangement in the environment.
  - We need to **plan movements of entities in time**, so that entities reach the goal arrangement while **physical limitations are observed**.
- **Physical limitations** are:
  - Entities must **not collide with each other**.
  - Entities must **not collide with obstacles** in the environment.
- Cooperative path-planning is also known as:
  - pebble motion on a graph
  - path-planning for multiple robots
- Certain slight variations in the definition allows higher parallelism

CPP – Formal definition (1)

Wilson, 1974; Kornhauser et al., 1984; Ryan, 2008

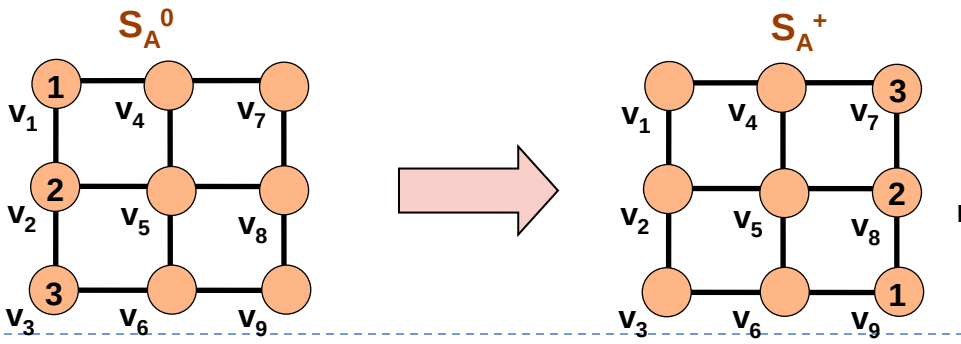
- A popular moving puzzle, that can be abstracted as the problem of cooperative path-planning is known as **Lloyd's fifteen.**
  - Entities are represented by **pebbles/agents** labeled by numbers.

- The environment is modeled as an **undirected graph** where **vertices represent locations** in the environment occupied by agents and **edges** enable agents to go to the **neighboring location**.
- **Formal definition** of the task of CPP
  - It is a quadruple $\Pi = (G, A, S_A^0, S_A^+)$, where:
    - $G=(V,E)$ is an **undirected graph**,
    - $A = \{a_1,a_2,...,a_\mu\}$, where $\mu<|V|$ is a **set of agents**,
    - S_A^0: $A \rightarrow V$ is a uniquely invertible function determining the **initial arrangement of agents** in vertices of G, and
    - S_A^+: $A \rightarrow V$ is a uniquely invertible function determining the **goal arrangement of agents** in vertices of G.

CPP – Formal Definition (2)

Wilson, 1974; Kornhauser et al., 1984; Ryan, 2008

- Time is discrete in the model. **Time steps** and their ordering is isomorphic to the structure of natural numbers.
- The **dynamicity** of the task is as follows:
  - An agent occupying a vertex at time step *i* can move into a neighboring vertex (the move is finished at time step *i+1*) if the target vertex is **unoccupied** at time step *i* and **no other agent** is moving simultaneously into the same target vertex
- For the given $\Pi = (G, A, S_A^0, S_A^+)$, we need to find:
  - A sequence of moves for every agent such that dynamicity constraint is satisfied and every agent reaches its goal vertex.



Solution of an instance of cooperative path-planning on a graph with A={1,2,3}

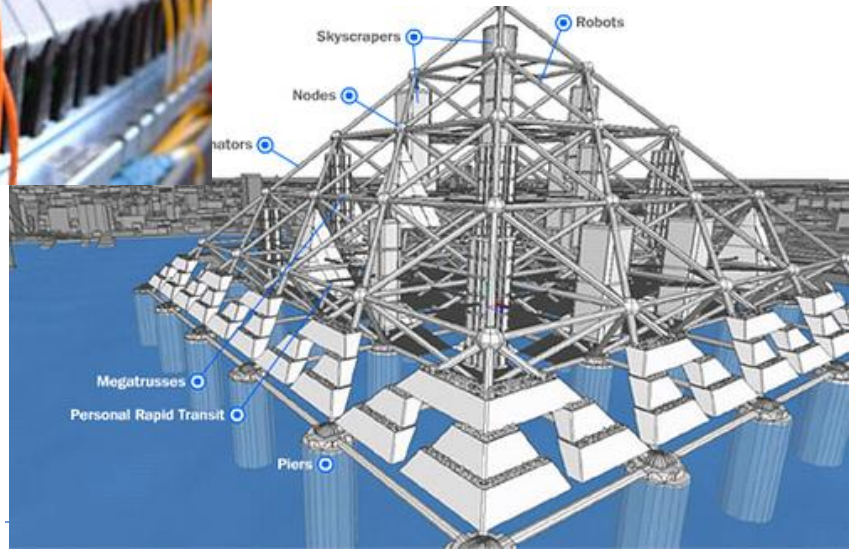
makespan=7

$\mathbf{M_1}=[\mathbf{v_1}, v_4, v_7, v_8, v_9, v_9, \mathbf{v_9}]$
$\mathbf{M_2}=[\mathbf{v_2}, v_2, v_1, v_4, v_7, v_8, \mathbf{v_8}]$
$\mathbf{M_3}=[\mathbf{v_3}, v_3, v_3, v_2, v_1, v_4, \mathbf{v_7}]$

Time step: **1 2 3 4 5 6 7**

Motivation

- Container rearrangement (entity = **container**)
- Heavy traffic (entity = **automobile** (in jam))
- Data transfer (entity = **data packet**)
- Generalized lifts (entity = **lift**)

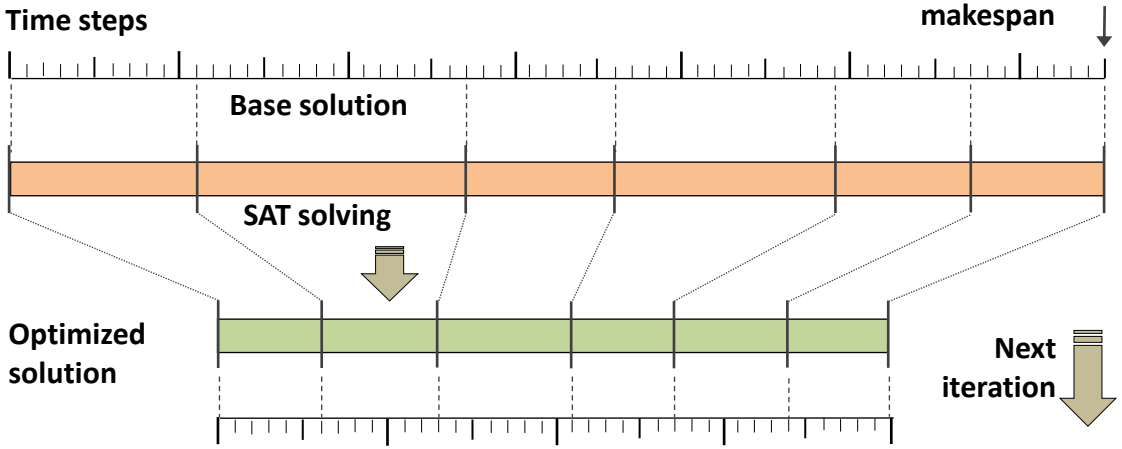


Is the task of CPP **easy** or **hard**?

- **Basic** variant of the task is **easy to solve** (makespan sub-optimal solution):
  - There exists an algorithm with **worst case time complexity** of $O(|V|^3)$ that generates solutions of the **makespan** $O(|V|^3)$ for any instance of CPP on G=(V,E) (Kornhauser et al., 1984).
- If we want a **solution** that has the **makespan as short as possible** the complexity increases:
  - The optimization variant of the CPP problem is **NP-hard** (Ratner a Warmuth, 1986)
  - Shown for the generalized **Lloyd's** puzzle (known as (N^2-1)-puzzle).
- We focused on generating and improving **sub-optimal** solutions towards optimal makespan

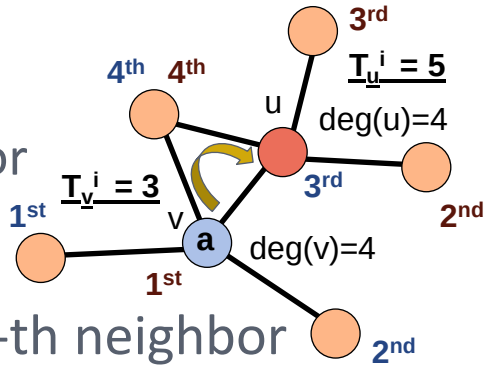
COBOPT – CPP as Propositional Satisfiability

- Suppose that we are able to construct a **propositional formula** such that
  - It is **satisfiable** iff there exists a solution to CPP of a given **makespan**
- Suppose that we are provided with **makespan suboptimal solution** (base solution – can be generated in polynomial time)
  - we can find **makespan optimal** replacement of the given sub-sequence of the base solution using:
    - propositional satisfiability solving + binary search (or some other type of search where query = SAT solving for the given makespan)



Inverse Encoding of CPP

- Makespan **m** is given
  - encode states of the planning world at time steps 1,2,…,m
  - state at the time **step 1** is enforced to be equal to the **initial state**
  - state at the time **step m** is enforced to be equal to the **goal state**
- Inverse encoding encodes **"what agent** is located in the given vertex"
- The state at the given time step *i* is described by the following **integer variables** for each $v \in V$:
  - $A_v^i \in \{0,1,2,\ldots, \mu\}$ with the interpretation that
    $A_v^i = j$ iff the agent a_j is located in v at the time step i or
    $A_v^i = 0$ iff there is no agent in v
  - $T_v^i \in \{0,1,2,\ldots, 2deg(v)\}$ with the interpretation that
    $0 < T_v^i \leq deg(v)$ iff the agent **goes out** of v into (T_v^i)-th neighbor
    $deg(v) \leq T_v^i \leq 2deg(v)$ iff the agent **goes into** v from $((T_v^i)$-$deg(v))$-th neighbor
    $T_v^i = 0$ iff no-operation is selected for v
- Plus constraints to enforce valid transitions between states



Translating to Propositional Satisfiability

- Each integer variable is encoded as a bit-vector where each bit is represented by a propositional variable
  - for example $A_v^i \in \{0,1,2,\ldots,\mu\}$ is encoded using $\lceil \log_2(\mu+1) \rceil$ propositional variables
  - extra states induced by the upper integer part are forbidden
- **Notice**: a bit-vector must take some of the values from its domain
  - each T_v^i must be assigned a value … in each vertex it must be decided what action is selected (no-op, incoming, outgoing)
  - ensures that agents do not collide with each other and maintains the frame
- Implications of the form $T_v^i = \text{constant} \Rightarrow A_u^{i+1} = \text{constant}$
  - translated using auxiliary propositional variables

All-Different Encoding of CPP

- If the environment contains few agents relatively to its size
  - inverse encoding contains lot of variables for empty space
- All-Different encoding encodes "**where is the given agent"**
- The state at the given time step *i* is described by the following **integer variables** for each $a \in A$:
  - $L_a^i \in \{1,2,\ldots, |V|\}$ with the interpretation that $L_a^i = j$ iff the agent a is located in the j-th vertex of the graph G
- The requirement that there is at most one agent per vertex is modeled as All-Different($L_{a1}^i, L_{a2}^i, \ldots, L_{a\mu}^i$)
- Other constraints are more complicated
  - it is necessary to express that agents can move along edges only
  - and that target vertex of the movement must be empty
- Augmenting by heuristics
  - some vertices are unreachable by the agent in the given time step

Encoding Size Comparison

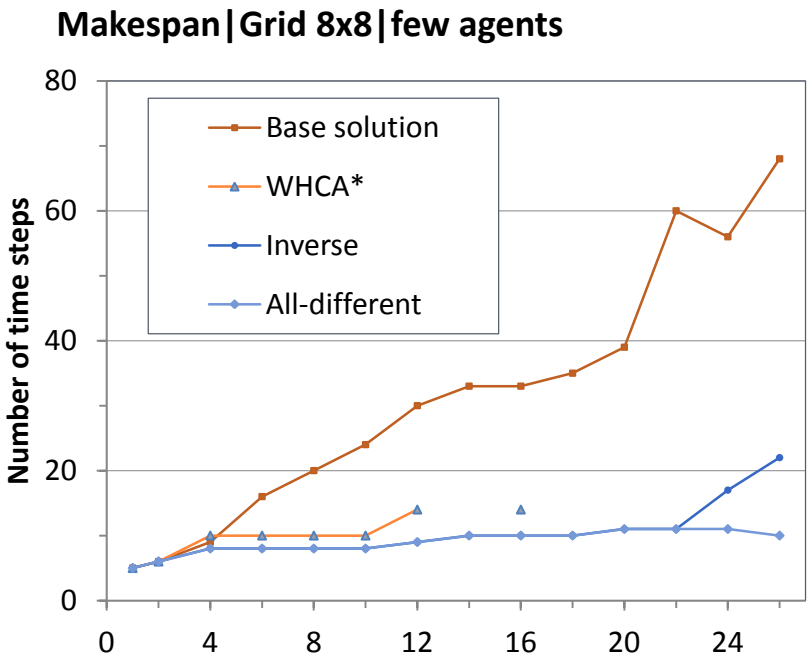
- Two setups grid of size 8x8 and 16x16
  - random initial and goal arrangement of agents

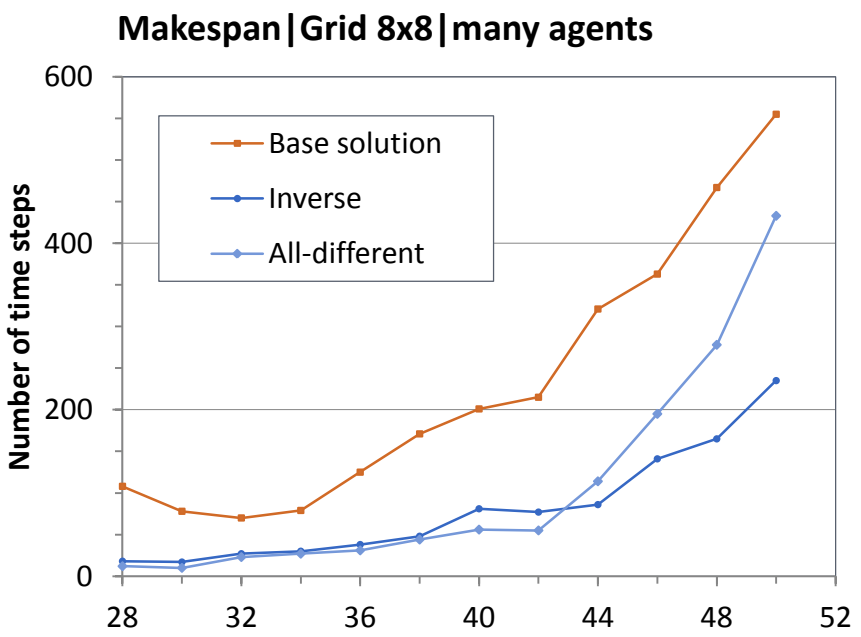
|A| in the 4-connected grid **8×8**	Number of **layers**	**SATPLAN** encoding		**SASE** encoding		**INVERSE** encoding		**ALL-DIFFERENT** encoding	
		|Variables|	**|Clauses|**	**|Variables|**	**|Clauses|**	**|Variables|**	**|Clauses|**	**|Variables|**	**|Clauses|**
4	8	5864	55330	11386	53143	5400	38800	11128	54356
8	8	10022	165660	19097	105724	5920	48224	25136	114952
12	8	14471	356410	26857	168875	5920	46176	42024	181788
16	10	30157	1169198	51662	372140	8122	76192	79008	326736
24	10	43451	2473813	73101	588886	8122	71072	140400	537528
32	14	99398	8530312	157083	1385010	12396	137120	309824	1120672

|A| in the 4-connected grid **16×16**	Number of **layers**	**SATPLAN** encoding		**SASE** encoding		**INVERSE** encoding		**ALL-DIFFERENT** encoding	
		|Variables|	**|Clauses|**	**|Variables|**	**|Clauses|**	**|Variables|**	**|Clauses|**	**|Variables|**	**|Clauses|**
4	21	69704	746562	137406	677737	60755	478462	122368	827628
8	15	65365	995507	134482	712352	46904	412416	178816	1174616
16	18	Out of memory		342100	2347456	61154	611328	469888	2928336
32	4*			288498	2716096	13672	143104	197888	1101600
40	4*			357762	3783672	13672	134912	265280	1415080
64	4*			561210	5913320	14700	189440	510464	2446912

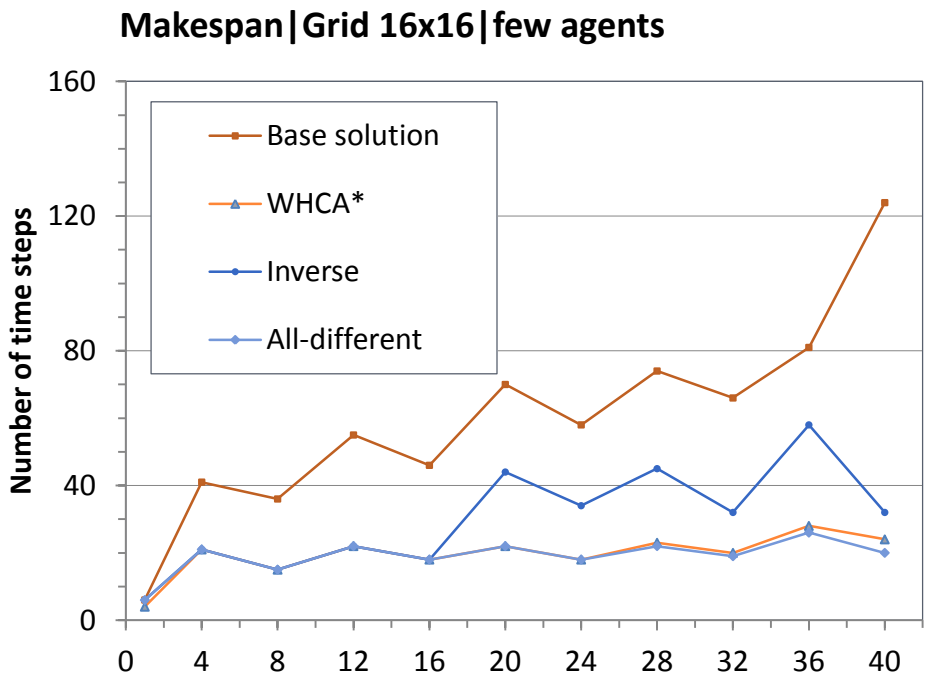
Makespan Comparison – grid 8x8

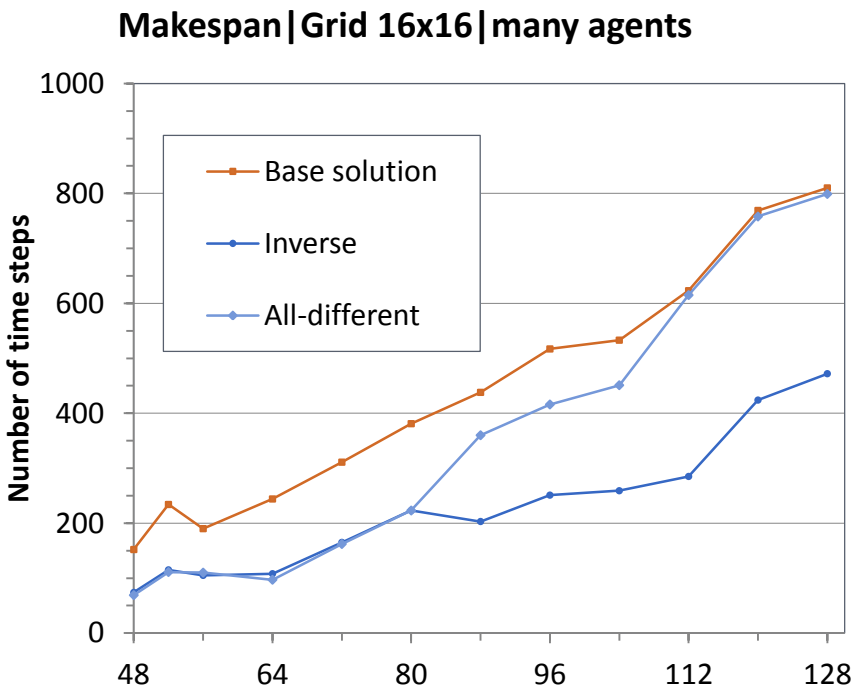
- Compared against WHCA*
  - WHCA* is decoupled
    - often produces near makespan optimal



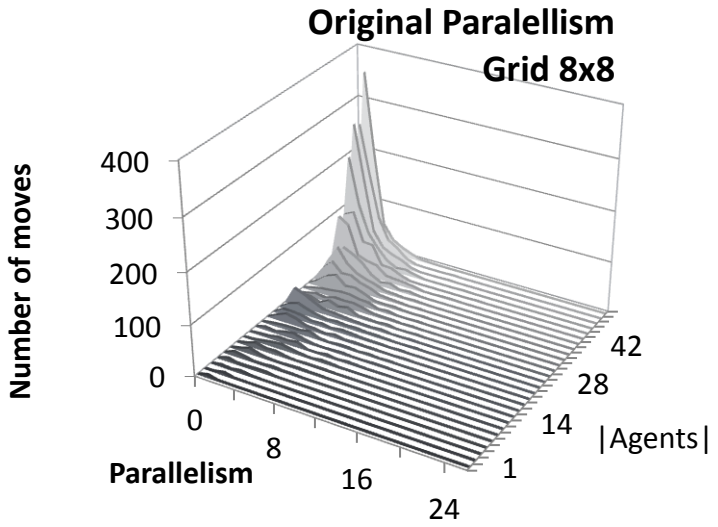


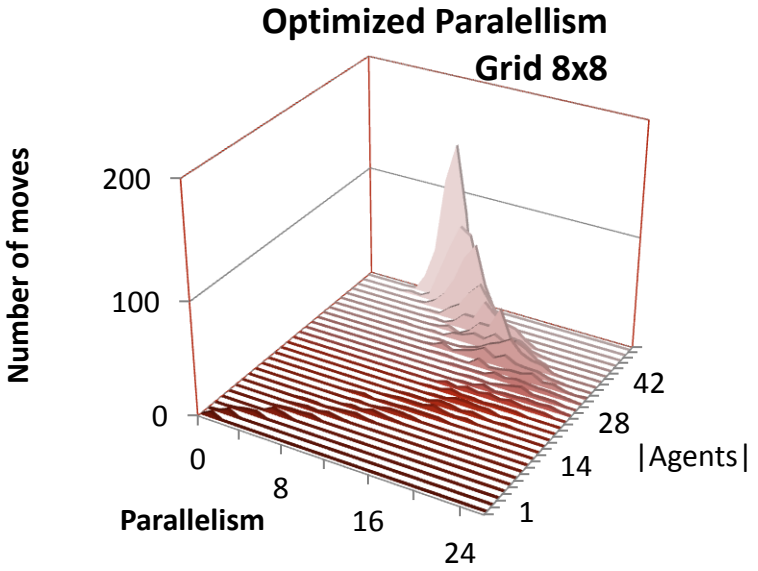
Makespan Comparison – grid 16x16

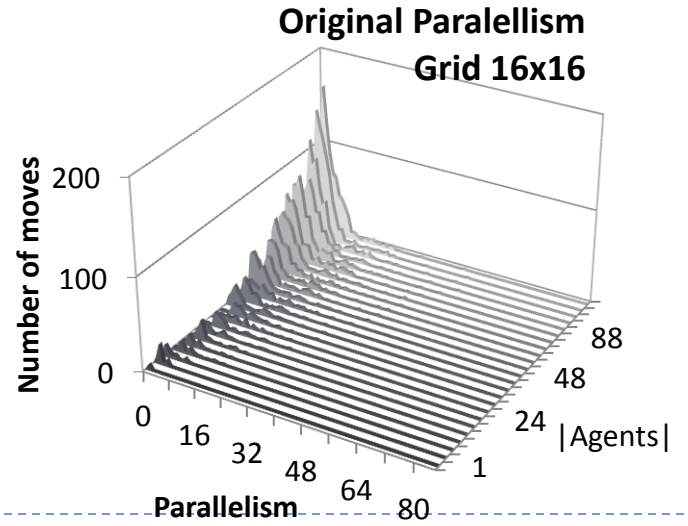


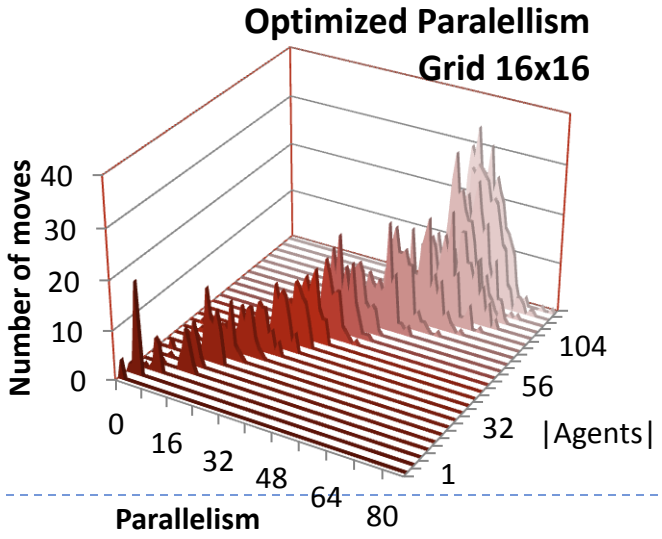


Parallelism Increasing









Concluding Remarks

- Improving sub-optimal solutions of cooperative path-planning by modeling the problem as propositional satisfiability.
- COBOPT: short subsequences of a sub-optimal solution are replaced by the makespan optimal ones.
- Two encodings (and its variants)
  - Inverse encoding
    - better in densely populated environments
  - All-Different encoding
    - better in sparsely populated environments
- COBOPT solution optimization together with both encodings represents state-of-the-art in generating short solutions to CPP